Example $n = 2$

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right)$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Claim:

- $\underbrace{X \otimes X}_{\to\, g_1} |\Psi\rangle = |\Psi\rangle$
- $\underbrace{Z \otimes Z}_{\to\, g_2} |\Psi\rangle = |\Psi\rangle$

$$\Rightarrow \underbrace{g_1 g_2 g_2 g_1 \cdots g_2}_{\text{group generated by } g_1, g_2} |\Psi\rangle = |\Psi\rangle$$

$$\langle g_1, g_2 \rangle = S$$

$$g_1 \cdot g_2 = (X \otimes X)(Z \otimes Z)$$

$$= (XZ) \otimes (XZ)$$

$$= (-iY) \otimes (-iY)$$

$$= -Y \otimes Y$$

$$g_2 \cdot g_1 = \ldots = -Y \otimes Y$$

$S =$

$\mathbb{1} \otimes \mathbb{1} \rightsquigarrow g_1^0 \cdot g_2^0$ $\qquad g_1 = X \otimes X$

$X \otimes X \rightsquigarrow g_1^1 \cdot g_2^0$ $\qquad g_1^0 = \mathbb{1} \otimes \mathbb{1}$

$Z \otimes Z \rightsquigarrow g_1^0 \cdot g_2^1$

$-Y \otimes Y \rightsquigarrow g_1^1 \cdot g_2^1$

$$P_S = \frac{1}{4}\left(\mathbb{1} \otimes \mathbb{1} + X \otimes X + Z \otimes Z - Y \otimes Y\right)$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$\mathrm{tr}\left[|\phi^+\rangle\langle\phi^+|\, Z \otimes Z\right] = 1 \checkmark$$

$$\mathrm{tr}\left[|\phi^+\rangle\langle\phi^+|\, X \otimes \mathbb{1}\right] = 0$$

$Y \otimes \mathbb{1} \rightsquigarrow 0$

$Z \otimes \mathbb{1} \rightsquigarrow 0$

$$\rightsquigarrow \mathrm{tr}_2\left[|\phi^+\rangle\langle\phi^+|\right] = \rho_1 = \tfrac{1}{2}\mathbb{1} .$$

Tomorrow

- graph states
- error correction