MAKING QUANTUM STATES OF LIGHT

1. Photons
2. Biphotoons
3. Squeezed states
4. Beam splitter
5. Conditional measurements
Beam splitter transformation
(Heisenberg picture)

• Quadrature transformation

\[
\begin{pmatrix}
\hat{a}_1 \\
\hat{a}_2
\end{pmatrix} =
\begin{pmatrix}
t & r \\
-r & t
\end{pmatrix}
\begin{pmatrix}
\hat{a}_1' \\
\hat{a}_2'
\end{pmatrix}
\]

where \( t^2 \) is the beam splitter transmission, \( r^2 \) reflection. \(|t|^2 + |r|^2 = 1\).

• If \( t \) and \( r \) are real, we can assign \( t = \cos \theta; \ r = \sin \theta; \)

\[
\begin{pmatrix}
\hat{a}_1 \\
\hat{a}_2
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\hat{a}_1' \\
\hat{a}_2'
\end{pmatrix}
\]

Also valid for positions, momenta

\[
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
x'_1 \\
x'_2
\end{pmatrix}, \quad
\begin{pmatrix}
p_1 \\
p_2
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
p'_1 \\
p'_2
\end{pmatrix}
\]

→ Beam splitter transformation = rotation in the phase space ⇒ entanglement

Problem: Show that a beam splitter acting on a pair of coherent states will generate a pair of coherent states
Example:
Einstein-Podolsky-Rosen state

- **State preparation**
  - Overlap $X$-squeezed $(\langle \delta x_1^2 \rangle < 1/2)$ and $P$-squeezed $(\langle \delta p_2^2 \rangle < 1/2)$ vacuum states on a symmetric beam splitter.
  - Beam splitter transformation:
    \[
    \begin{pmatrix}
    X_1 \\
    X_2
    \end{pmatrix}
    \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix}
    X_2 - X_1 \\
    X_2 + X_1
    \end{pmatrix};
    \begin{pmatrix}
    P_1 \\
    P_2
    \end{pmatrix}
    \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix}
    P_2 - P_1 \\
    P_2 + P_1
    \end{pmatrix}
    \]
  - After beam splitter:
    \[
    \langle \delta(x_1 - x_2)^2 \rangle < 1/2; \langle \delta(p_1 + p_2)^2 \rangle < 1/2
    \]
  - Both positions and momenta are nonclassically correlated.
  - Entangled state generated!
  - This states approximates the original Einstein-Podolsky-Rosen state.
By the way:
the original EPR paradox

**The ideal Einstein-Podolsky-Rosen state**

- In position representation: \( \Psi_{\text{EPR}}(x_1, x_2) = \delta(x_1 - x_2) \)
- In momentum representation: \( \Psi_{\text{EPR}}(p_1, p_2) = \delta(p_1 + p_2) \)

Problem: obtain the position representation wave function from the momentum representation.

**If shared between Alice and Bob:**

- If Alice measures position \( x_1 \)
  \( \rightarrow \) Bob receives a position eigenstate \( |x_2\rangle = |x_1\rangle \)
- If Alice measures momentum \( p_1 \)
  \( \rightarrow \) Bob receives a momentum eigenstate \( |p_2\rangle = |-p_1\rangle \)

**Alice can create two mutually incompatible physical realities at a remote location**

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Theory: Einstein, Podolsky, Rosen, PRA **47**, 777 (1935)
Experiment: Z.Y.Ou et al., PRL 68, 3663 (1992)
Beam splitter transformation (Schrödinger picture)

- Photon number transformation

\[ |m,n\rangle \rightarrow \sum_{j,k=0}^{m,n} \sqrt{(j+k)!(m+n-j-k)!} \begin{pmatrix} m \\ j \end{pmatrix} \begin{pmatrix} n \\ k \end{pmatrix} (-1)^k t^{n+j-k} r^{m-j+k} |m+n-j-k,j+k\rangle \]

- Simplest example: splitting a photon

\[ |1\rangle \rightarrow t |1\rangle_A |0\rangle_B - r |0\rangle_A |1\rangle_B \]
Example: Tomography of a dual-rail qubit

- Photon hits a beam splitter → a two-mode qubit is generated
  \[ |1\rangle \rightarrow t|1\rangle_A|0\rangle_B - r|0\rangle_A|1\rangle_B \]
- Measure quadratures \( X_A \) and \( X_B \) via homodyne detectors
- Phase/dependent quadrature statistics → state reconstruction

Example: Tomography of a dual-rail qubit (…continued)

- Probability distributions $\text{pr}(X_A, X_B)$
  - Serve as marginal distributions for the 4-D Wigner function
  - Entanglement $\rightarrow$ Nonclassical, phase-dependent correlations

\[ \theta_A - \theta_B = 0 \quad \quad \theta_A - \theta_B = \frac{\pi}{2} \quad \quad \theta_A - \theta_B = \pi \]

- Probability distributions $\text{pr}(X_A, X_B)$ for all $\theta_A, \theta_B$
  $\rightarrow$ quantum state reconstruction
Why do we see such distributions?

- If no beam splitter is present

$|1\rangle$ → $\theta_A - \theta_B$

$|0\rangle$ → $X_{\theta_A}$, $X_{\theta_B}$

- Alice measures the single-photon state, Bob measures the vacuum state
- Measurements are uncorrelated → distributions are uncorrelated
- No entanglement
- No information about relative phase $\theta_A - \theta_B$
Why do we see such distributions?

- If beam splitter is present

\[
|1\rangle \quad |0\rangle
\]

- Alice and Bob measure the same quadrature \((\theta_A - \theta_B = 0 \text{ or } \pi)\)
  \(\rightarrow\) uncorrelated distribution rotates by 45°
- Alice and Bob measure different quadratures \((\theta_A - \theta_B = \pi/2)\)
  \(\rightarrow\) distribution remains uncorrelated
Tomography of an optical qubit

Results


- Density matrix
  - Serve as marginal distributions for the 4-D Wigner function
  - Entanglement $\rightarrow$ Nonclassical, phase-dependent correlations

First complete (not postselected) reconstruction of an optical qubit
Another example: Hong-Ou-Mandel dip

- Two photons "colliding" on a beam splitter will stick together

\[ |1,1\rangle \rightarrow (|2,0\rangle - |0,2\rangle)/\sqrt{2} \]

- Hong-Ou-Mandel effect: correlation count in the beam splitter output vanishes when the two photons arrive simultaneously.

Hong, Ou, Mandel, PRL 59, 2044 (1987)
Beam splitter model of absorption

- The problem
  - A quantum state of light propagates through an attenuator. What is the transmitted state?

\[ |\psi_{in}\rangle \quad \hat{\rho}_{out} = ? \]
Beam splitter model of absorption

• **The solution**
  - Replace the absorber with a beam splitter.
  - The second input is vacuum

\[ |\psi_{in}\rangle \]

\[ |\Psi\rangle = \hat{B} |\psi_{in}\rangle |0\rangle \]

• Beam splitter output \(|\Psi\rangle\) may be entangled
• Mode 2 in the beam splitter output is lost
• To find \(\hat{\rho}_{out}\), trace over the lost mode in the beam splitter output

\[ \hat{\rho}_{out} = \text{Tr}_2 |\Psi\rangle\langle\Psi| \]
Beam splitter model of absorption

- **The solution**
  - Replace the absorber with a beam splitter.
  - The second input is vacuum

\[
|\psi_{in}\rangle
\]

\[
|0\rangle
\]

\[
|\Psi\rangle = \hat{B}|\psi_{in}\rangle|0\rangle
\]

- **In terms of Wigner functions**
  - Beam splitter input Wigner function: \( W_{|\psi\rangle|0\rangle} = W_{|\psi\rangle}(x_1, p_1)W_{|0\rangle}(x_2, p_2) \).
  - To find the beam splitter output Wigner function \( W_{|\Psi\rangle}(x_1, p_1, x_2, p_2) \) apply phase-space rotation.
  - To find the Wigner function of mode 1, integrate over mode 2:

\[
W_{out}(x_1, p_1) = \int_{-\infty}^{+\infty} W_{|\Psi\rangle}(x_1, p_1, x_2, p_2) dx_2 dp_2.
\]
MAKING QUANTUM STATES OF LIGHT

1. Photons
2. Biphotons
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5. Conditional measurements
Conditional preparation of a photon

- **Parametric down-conversion**
  - “Red” photons are always born in pairs
  - Photon detection in one emission channel
    → there must be a photon in the other channel as well

Not a single photon “on demand”

To date, this is the only method which provides a single photon with a high efficiency in a certain spatiotemporal mode
**Schrödinger cat**

What does it mean in optics?

• **Coherent superposition of two coherent states**

\[ |\text{cat}_\pm\rangle = |\alpha\rangle \pm |-\alpha\rangle \]

• Useful for quantum teleportation quantum computation, and error correction

• Fundamentally important

**Problem. Calculate these Wigner functions**

• **Compare: incoherent superposition of two coherent states**

\[ \hat{\rho} = |\alpha\rangle \langle \alpha| \pm |-\alpha\rangle \langle -\alpha| \]

• Boring, classical state
Schrödinger cat
How to make one?

• Easy for small $\alpha$’s

\[
|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle + \alpha_3 |3\rangle + \ldots
\]

\[
|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle + \alpha_3 |3\rangle + \ldots
\]

\[
|\text{cat}_-\rangle = \alpha_1 |1\rangle - \alpha_3 |3\rangle + \ldots \quad \text{...just a squeezed single-photon state!}
\]

\[
\alpha = 2.1 \\
\alpha = 1.4 \\
\alpha = 0.7
\]
Schrödinger cat

How to make one?

- **Making a squeezed single-photon state**
  - Create a squeezed state
    \[ |\psi_s\rangle = \beta_0 |0\rangle + \beta_2 |2\rangle + \beta_4 |4\rangle + \ldots \]
  - Subtract a photon
    \[ \hat{a} |\psi_s\rangle = \sqrt{2} \beta_2 |1\rangle + 2 \beta_4 |3\rangle + \ldots \]

[Reproduced from A. Ourjoumtsev et al., Science 312, 83 (2006)]


K. Wakui et al., quant-ph/0609153
Summary to part 2: Classification of quantum state preparation methods

- **“On demand”:**
  State is readily available when required by the user
  Example: photon from a quantum dot

- **“Heralded”:**
  State produced randomly; system provides user with a classical signal when the state is produced
  Example: heralded single photon

- **“Postselected”:**
  State is not known to have been produced until it is detected
  Example: photon pair from a down-converter

Postselected + conditional measurement = Heralded (maybe)
Heralded + memory = On demand
QUANTUM REPEATER

and memory for light
Quantum cryptography: here and now

Secure communication up to 100-150 km
- Free space
- Optical fibers

Commercialization begins
- Id Quantique (Switzerland)
- MagiQ (Boston)
- BBN Technologies (Boston)

Metropolitan quantum communication networks
- Geneva
- Boston
- Vienna
- Calgary
Problems with quantum cryptography

- Preparation of single photons
  - Must ensure absence of two-photon pulses
- Losses in optical fibers
  - 0.2-0.3 dB/km: half of photons are lost over 10-15 kilometers.
  - Example: Dubai to Kish, 300 km, only 1 in 30,000,000 photons will reach destination
  - Can't use amplifiers
- "Dark counts" of detectors
  - Sometimes a photon detector will "click" without a photon
  - Dark clicks cause errors
  - Too many errors → can't detect eavesdropping
Suppose Alice wants to send a photon to Bob...

The photon is likely to get lost on its way
Quantum relay

• If Alice and Bob shared an entangled resource,
  - Alice could *teleport* her photon to Bob
  - But long-distance entanglement is difficult to create
Quantum relay

Long-distance entanglement can be created by entanglement swapping

A Bell measurements on modes 2 and 4 entangles modes 1 and 4
Long-distance entanglement can be created by *entanglement swapping* but to succeed, all links must work simultaneously.

→ success probability still decreases exponentially with distance.
The role of memory

- But if we had quantum memory,
  - entanglement in a link could be stored…
    until entanglement in other links has been created, too.
  - Bell-measurement on adjacent quantum memories…
    will create the desired long-distance entanglement.
  - Alice can teleport her photon to Bob
• **This technology is called quantum repeater**
  • Initial idea: H. Briegel *et al.*, 1998
  • In application to EIT and quantum memory: L.M. Duan *et al.*, 2001
• Quantum memory for light is essential for long-distance quantum communications.
By the way…

- Quantum memory for light is also useful in quantum computing
  - Photon makes an excellent qubit… but does not like to stay put
  - Any computer, quantum or classical, needs memory
ELECTROMAGNETICALLY INDUCED TRANSPARENCY and memory for light
What is EIT?

Quantum interference effect in atoms with Λ-shaped level structure

What will happen to the signal field when we send it through an EIT medium?
Absorption of the signal field

**Without control field**

Narrow transparency window on resonance.
- Light propagates through an otherwise opaque medium.
Dispersion of the signal field

We can enormously reduce the group velocity
• Group velocity is proportional to the control field intensity

\[ v_g(\omega) = \frac{c}{n(\omega) + \omega \frac{dn}{d\omega}} \]
EIT for quantum memory

- The idea
  - Turning the control field off will reduce the group velocity to zero

  - Quantum information contained in the pulse is stored in a collective atomic ground state superposition

  - Turning the control field back on will retrieve the pulse in the original quantum state
EIT for quantum memory
EIT in our lab

- Implementation in atomic rubidium
  - Ground level split into two hyperfine sublevels → a perfect Λ system
  - Control and signal lasers must be phase locked to each other at 6.8 GHz
EIT-based memory: practical limitations

- EIT window not perfectly transparent → part of the pulse will be absorbed
- Memory lifetime limited by atoms colliding, drifting in and out the interaction region
Storage of squeezed vacuum
The setup

Ti:Sapphire laser

6.8 GHz phase lock

Diode laser

Control field

Squeezed vacuum

AOM

Rb87 + Ne

Oven with cell

PBS

Chopper

OPA

SHG cavity

HD

LO
Storage of squeezed vacuum
The initial state

Quadrature data
Density matrix
Wigner function
Storage of squeezed vacuum
The retrieved state

- Maximum squeezing: 0.21±0.04 dB
- Squeezing observed in the retrieved state!
EIT for quantum memory: state of the art

The “holy grail”

- Store and retrieve arbitrary states of light for unlimited time
- State after retrieval must be identical to initial

Existing work

- L. Hau, 1999: slow light
- M. Fleischauer, M. Lukin, 2000: original theoretical idea for light storage
- M. Lukin, D. Wadsworth et al., 2001: storage and retrieval of a classical state
- A. Kuzmich et al., M. Lukin et al., 2005: storage and retrieval of single photons
- M. Kozuma et al., A. Lvovsky et al., 2007: memory for squeezed vacuum

Existing benchmarks

- Memory lifetime: up to milliseconds in rubidium, up to seconds in solids
- Memory efficiency: up to 50 % in rubidium, lower for solids
- Things get much worse when we attempt to store nonclassical states of light
QUANTUM COMPUTATION GATES

1. With EIT
2. Using conditional measurements
An optical C-NOT gate

• What we need

\[ |H\rangle \text{ or } |V\rangle \]

\[ |H\rangle \text{ or } |V\rangle \]

\[ \text{ENTANGLING GATE} \]

\[ |H\rangle |H\rangle \rightarrow |H\rangle |H\rangle \]

\[ |H\rangle |V\rangle \rightarrow |H\rangle |V\rangle \]

\[ |V\rangle |H\rangle \rightarrow |V\rangle |H\rangle \]

\[ |V\rangle |V\rangle \rightarrow -|V\rangle |V\rangle \]
An optical C-NOT gate

- **How to implement this**

- **Nonlinear phase shift**
  \[|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle\]
  \[|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle\]
  \[|1\rangle|0\rangle \rightarrow |1\rangle|0\rangle\]
  \[|1\rangle|1\rangle \rightarrow -|1\rangle|1\rangle\]

- **Problem**
  - No materials that exhibit optical nonlinearity at the single-photon intensity level
QUANTUM COMPUTATION GATES

1. With EIT

2. Using conditional measurements
Nonlinear optics with EIT

- Basic idea: exploit steep dispersion curve to produce large cross-phase modulation

- Small change in 2-photon detuning → Large change in transmitted signal phase
N-type scheme

• EIT on signal field due to $|1\rangle|2\rangle|3\rangle$ $|4\rangle$

N-scheme

- EIT on **signal field** due to $|1\rangle|2\rangle|3\rangle$

- Off-resonant coupling of weak **induction field** produces Stark shift on $|1\rangle$
  - Changes 2-photon detuning of signal EIT
  - Affects phase of transmitted signal

N-scheme
[continued]

• Problem with N-scheme:
  • Only signal field experiences slowdown.
  • For pulses, this is a severe limitation.

• Solution:
  • Slow down induction pulse via another EIT system
    • Lukin, Imamoglu (2001): use another atomic species (e.g. $^{85}$Rb)
    • Wang, Marzlin, Sanders (2006): use double EIT in the same atom
QUANTUM COMPUTATION GATES

1. With EIT

2. Using conditional measurements
Non-deterministic phase gate: implementation with the beam splitter

- General beam splitter transformation

\[ |m,n\rangle \rightarrow \sum_{j,k=0}^{m,n} \sqrt{(j+k)! (m+n-j-k)!} \binom{m}{j} \binom{n}{k} (-1)^k t^{n+j-k} r^{m-j+k} |m+n-j-k,j+k\rangle \]

Entangles input modes
Entanglement very complicated
Conditional measurement and/or postselection are required to implement computation gates

⇒ Linear-optical quantum computing is non-deterministic

Non-deterministic phase gate: implementation with the beam splitter

• Beam splitter with reflectivity $1/3$

\[
\begin{align*}
(r = \sqrt{\frac{1}{3}}, t = \sqrt{\frac{2}{3}}) \\
\end{align*}
\]

• Postselect on events in which the number of photons in the reflected channel is the same as that in the corresponding incident channel

• Neglect all other events

\[
\begin{align*}
|0,0\rangle &\rightarrow |0,0\rangle \\
|1,0\rangle &\rightarrow \frac{1}{3} |1,0\rangle \\
|0,1\rangle &\rightarrow -\frac{1}{3} |0,1\rangle \\
|1,1\rangle &\rightarrow \frac{1}{3} |1,1\rangle \\
\end{align*}
\]

Insert $\pi$ phase shift into the right channel

\[
\begin{align*}
|0,0\rangle &\rightarrow |0,0\rangle \\
|1,0\rangle &\rightarrow \frac{1}{3} |1,0\rangle \\
|0,1\rangle &\rightarrow \frac{1}{3} |0,1\rangle \\
|1,1\rangle &\rightarrow \frac{1}{3} |1,1\rangle \\
\end{align*}
\]

Phase gate implemented!

召回 Non-deterministic (probability = 1/3 per photon)

→ Need to attenuate horizontal photons, too
Non-deterministic phase gate
[continued]

• **Full scheme**

  - attenuator for horizontal photons (transmission = 1/3)
  - beam splitter for vertical photons
  - $\pi$ phase shift

• **Properties**

  - Works conditioned on detecting 1 photon in each output
  - Works with probability 1/9
  - Would be useful for quantum computing if we had non-demolition detection of photons
Non-deterministic phase gate
Experimental implementation

• The setup
  • Partially-polarizing beam splitters used to simplify mode-matching
  • Operation of the gate as a Bell-state analyzer verified

Another example: Conditional preparation of multi-photon entanglement

- **Greenberger-Horne-Zeilinger state**
  \[|HHV\rangle + |VVH\rangle\]

- **Conditioned on 4-photon coincidence** (postselected preparation)
  - Start from 2 pairs \(|HV\rangle - |VH\rangle\)
  - Photon that fires T comes from “first pair”
    - \(\Rightarrow\) first pair must be \(|HV\rangle\)
    - \(\Rightarrow\) second pair must be \(|VH\rangle\)
  - Photons transmitted and reflected from BS must be of opposite polarizations
  - Photons detected by \(D_1\) and \(D_2\) must be of the same polarization
  - The state incident on \(D_1, D_2\) and \(D_3\) is either \(|HHV\rangle\) or \(|VVH\rangle\)
  - These possibilities are indistinguishable
    \(\Rightarrow\) The output state is a coherent superposition

\(\Rightarrow\) We know the state has been generated only after it’s detected

*D. Bouwmeester et al., PRL 82, 1345 (1999)*
Thanks!

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  - CFI
  - QuantumWorks

Ph.D. positions available

http://qis.ucalgary.ca/quantech/